The practical geometry of Persian ribbed vaults: A study of the rehabilitation of the Kolahduzan Dome in the Tabriz historic bazaar

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ABSTRACT: Recognized as a UNESCO World Heritage site in 2010, the bazaar of Tabriz is one of the largest brick complexes in the world. In the mid-20th century, the complex came under the management of the Iranian Cultural Heritage, Handicrafts and Tourism Organization (ICHTO). This article focuses on the rehabilitation of the Kolahduzan Dome in the old fabric bazaar. By the 1970s, the dome was crumbling following decades of neglect. Conservation of the dome began in 1981 under the direction of Reza Memaran Bename Tabrizi, a member of the last generation of local traditional builders. This article delves into the geometrical analysis of the new design and the construction technique adopted by the master builder. The geometrical analysis provides a new perspective on practical geometry in Persian vaulting. It identifies three geographically different layers: 1. Theoretical geometry, 2. Structural geometry, and 3. Architectural geometry. On-site data collection was performed by the author to record all workable points on the vault. Historical documents have been received from Iran Cultural Heritage Documentation Center.

1 INTRODUCTION
This research investigates one of the monuments built by the master [Ustād] Reza Memaran Bename Tabrizi over the Charsough-i Kolahduzan in the historical complex of the bazaar of Tabriz. The Kolahduzan Dome was redesigned and reconstructed by Us Reza in 1982. Little is known about the original vault and covering of the site. However, interviews with older shop owners in the area reveal that the cupola’s original roofing material was lost to untreated earthquake damage and lack of timely restoration. Us Reza was able to reconstruct the dome using solely traditional method. This method, known as kārbandī in Iran, utilizes load-bearing intersecting ribs. kārbandī is a traditional Iranian architecture used to cover large-scale openings in public spaces. This method employs intersecting stellar lines and theoretical mathematical principles.

What supplies the unifying force to Islamic architecture is the geometrization of design, structure, ornament, and space (Golombek & Wilber 1988). Geometry is as much a theme of the architecture as it is of the decoration. The first examples of this kind of stellar vault to be identified by Persian architects as kārbandī appeared in Umayyad Cordoba in the 10th century. Over the next two centuries this technique appears to have made its way to some of the Islamicate world’s eastern cities like Nā’in and Merv. The latter is shown over the tomb of Sultan Ahmad Sanjar (Nazari & Hashemi Nik 2019, 7).

Studying the design of the diminutive cupola of Kolahduzan revealed several keys to understanding the approach and methods of Us Reza. His approach to aesthetics and structural design and his personal affinity for embedded ribs (taqdozd) are revealed through the examination of the cupola. Embedded ribs are hidden load-bearing ribs that function as a main design element of a vault.

Because the Kolahduzan Dome was reconstructed 35 years ago by a contemporary Iranian architect trained in traditional methods, and there are compelling documents in the archive of ICHTO (Iranian Cultural Heritage, Handicrafts and Tourism Organization) concerning the dome’s construction process, this cupola’s reconstruction represents an important case study in the use of practical geometry in Persian architecture. This article presents the results of a study to examine the architectural approach to the geometrical design of the dome’s skeleton, the structural role of the transverses and tiercerons, and use of embedded ribs. Before proceeding with the discussion of the structure’s technical elements, it is important to provide a glimpse of the history of Tabriz and its bazaar.

2 THE HISTORICAL BAZAAR OF TABRIZ
Tabriz (north-west, Iran) is the Iran’s sixth largest city, and it has played an important role in the historical, cultural and political facets of the contemporary history of Iran. Owning to the fact that the city hosted the residence of the crown prince during the Qajar era (1789–1925), Tabriz became the center of the Iranian Constitutional Revolution between 1905 and 1911. During the Qajar era, Tehran became the political capital of Iran. However, Tehran’s economic growth was delayed due to Tabriz’s importance as the home of First
Chief of the Persian Chamber of Commerce, Malek al-Tojjar Aqa Mehdi Tabrizi who was instrumental in expanding and improving the bazaar of Tehran (Farmanfarmaian 2012, 203). The task of transforming the bazaar was left to Amir Kabir (d. 1851), the reformist prime minister of Nasir al-Din Shah, and Aqâ Mehdi Tabrizi Malek al-Tojjar (King of the Merchants of the Protected Domains of Iran).

The result was a vast program of restoration and expansion of the Tehran Bazaar (Keshavarzian 2007, 22). The monumental bazaar has been a symbol of Tabriz’s greatness throughout the centuries. In 1330, the famous Moroccan explorer Ibn Battuta wrote that the bazaar of Tabriz was one of the largest bazaars in Muslim countries (Gibb 1929, 102). Marco Polo and the Spanish traveler, Clavijo, wrote about the architectural glories and economic prosperity of the complex in 1275 and 1403 respectively. The bazaar’s current architectural structure is 230 years old.

The bazaar’s earlier structure had been almost completely demolished by the disastrous earthquake of 1780. The complex (Figure 1) has an organic fabric, wherein a system of land-use and geographic configuration develops naturally out of the interplay of market forces. The main commercial components are the configuration axes (rasteh-e-ha), the chambers (hojreh-ha), the caravanserais, and the recessed domed vestibular intersections (charsough). Spatial models of charsoughs usually contain a covered central court around which commercial cloisters and axes are arranged. The axes are vaulted vestibules, made up of vaulted vestibules, with a length of 5.5 km and width varying between four and five meters, and flanking chambers. Most of the flanking chambers have two stories. The ground floor is used as a trading space and the second-floor rooms are used for storage.

3 REZA MEMARAN BENAM TABRIZI

Ustâd (Us) Reza (1903–95) was one of the most famous Iranian contemporary architects of the last century. He was among the last in a line of local traditional builders in Iran, such as Hossein Lorzadeh (1906–2005) and Ali Asgar She’r’raf (1931–2016).

Us Reza began his architectural training at the age of 20 under the supervision of his father, Us Balakazem. In an interview with Dr. Farhad Tehrani, Us Reza notes “I had been studying theology before learning architecture, because the study of theology was not suitable for my personality, at the age of 20, I began architecture” (Tehrani interview 1981).

Us Reza’s name emerged as designer and builder not only of some of the most important projects by ICHTO, like the rehabilitation project of Masjid-i Kabud’s central dome in Tabriz (1975), but also in a number of major projects in Tehran, such as the Masjid-i Sharif University dome (Figure 2), all using traditional construction methods. He was well-known for his vaulting techniques and use of geometrically intricate crossed-arch structures for the bridging of large spans.

Due to the rise in the use of modern construction methods in Iran in the 1960s, traditional architects were no longer in demand. In the 1970s, the establishment of ICHTO and developments in the science of historic preservation, led to the documentation of the professional knowledge of traditional architects, like Us Reza, through their cooperation in restoration projects. The following section presents an analysis of the structure and the geometry on the stellar vault composition of the Kolahduzan Charsough. This analysis was carried out using the following methods: surveying the structure’s present condition, regenerating the plan of the ribs, consulting archival documents from the Iran Cultural Heritage Documentation Center, and using Rhinoceros 6 to produce the 3D model of the intersecting arches. In this article, I shall also
address the question of whether traditional master builders based their intersecting vaults on geometrical concepts. Firstly, the geometrical knowledge of the architect behind the design of the ribs shall be investigated. Secondly, the stages of the construction process corresponding to the transverses, tiercerons and liernes will be described. Finally, the construction technique employed in the assembly of the ribs will be described.

4 GEOMETRICAL ANALYSIS

4.1 The arch profile

In any geometric analysis of a vault built on pointed arches, the identification of the arch profile and the geometrical knowledge behind the design must be determined. Unlike in the case of semicircular or oval arches, the possibilities of the pointed arch are endless due to the possible width/rise ratios determined by the locations of the arc centers. Multi-centered arches in Islamic architecture, recognizing Persian influence to various degrees, can be grouped into recognizable sets. Most pointed arches in Iranian sphere are three- or four-centered. The origins of the four-centered pointed arch demand a thorough investigation. This model made its debut under the Il-Khanids (1256–1335). A surge in design experiments related to this model occurred in the 14th and 15th centuries.

Four particular profiles of the Kolahduzan Dome are of interest, namely the load-bearing transverses (Figure 5). Since the four transverses are assumed to be equal, only one transverse was scanned. The vault was surveyed using a Leica FlexLine TS 06 which recorded points and vertices. The first plotting attempt revealed that the geometry of the profile is quite similar to the most well-known Persian four-centered pointed arch, colloquially known as panjo haft (literally translates to: five-and-seven). The geometry of the arch profile is illustrated in Figure 6.

4.2 Theoretical geometry

Unlike free-handed vaults, ribbed vaults (kārbandi) conform to strict mathematical principles. Understanding these principles will illuminate the design process behind these ribbed vault structures. The construction methods including projection of a star polygon from the plane onto the rotating surface of a solid (kārbandi) is a significant pattern in the Iranian architecture vaulting (Hashemi Nik 2013, 11) that are designed based on a series of theoretical principles of mathematics, known as “Theoretical Geometry.” Before introducing the theoretical geometry of the Kolahduzan Dome, it should be mentioned that Al-Fārābī identified seven fields of mathematics. Each of these mathematical fields had a branch of theory and a branch of practice. Carpenters, masons and metal workers deal with the practical geometry as defined by al-Fārābī (Necipoğlu 2017, 20).

In al-Farabi’s classification system, theoretical geometry contains definitions, theorems and proofs. He describes practical geometry as “looking into the lines and surfaces, within a wooden body if it is used by a joiner (najjar), and within a wall if it is used by a master builder (bānū), or within surfaces of land and fields if it is used by a surveyor (māshīh). Al-Farabi and Abu al-Hasan al-?Amiri (d. 992) also identified another branch of mathematics within the branch of geometry which they called, “The Science of Building Engineering” (ilm al-hiyal) (Necipoglu 2017, 20) and it refers to searching for ways for human beings to impose mathematical theories on external objects (Figure 4).

The architectural plan of the charsough is a semi-oblong octagon, its longer sides measure 4.6 meters, opening directly into the main axes (rasteh-ha), and its shorter sides measure 2.5 meters. The cupola over the hall is about 9.5 meters high.

It is segmented by four transverses and secondary and tertiary ribs: tiercerons and liernes.
The key to understanding the composition of the intersecting ribs is to divide an imaginary circle circumscribed around an octagon (here the octagon represents the charsough plan) into twelve equal arcs, then connect the chords four in between. Assuming this imaginary circle was the modular format on which the vault had been developed facilitates an understanding of the morphology of the composition of the ribs. Therefore, the theoretical geometry of the design can be described in the following way: if the points created by dividing the circumscribed circle around the octagon into twelve equal parts, are connected four in between, a star-shaped polygon, known as “kārbandī/rasmī {12/4}” in Iranian architectural terminology, emerges (Figure 3).

Here the nomenclature is purely mathematical. The first integer denotes the number of vertices on the represented polygon (12), and the second denotes the connection sequence between said vertices (4). The star-polygon shapes a twelve-sided central rosette (numbered consecutively from 1 to 12 in Figure 3), known in Iranian art and crafts as Shamseh. The number of the kārbandī is usually displayed in the extremities of its rosette.

The finished geometry (the theoretical geometry) is the horizontal image of the intersecting ribs on the ground whose convex vertices have an internal angle of 30 degrees. In a practical experience, each chord represents a 2D pointed-arch in front view (Figure 6), and a solid, masonry pointed-arch in the finished vault. The whole geometry represents a three-dimensional network of intersecting ribs.

4.3 From theory to practice

Structural and decorative functions, construction techniques, and applied geometry in rib-vaulting, particularly load-bearing capacity of the masonry vaults, would make an interesting subject in the study of construction history. Ever since foreign scholars like Arthur Upham Pope, André Godard, and Donald Wilber wrote about Iranian architecture in the early 20th century, scholars interested in Iranian architecture have engaged in speculation and debate about a variety of matters pertaining to the topic.

Referring to the previous argument, theoretical geometry was the basis for the structural design of the Kolahduzan Dome. Hence, Us Reza purposefully selected and applied the kārbandī {12/4} and technical considerations to accommodate the boundary conditions of the site. A point of interest here is that under certain situations, architects eliminate or trim some lines parts of the theoretical geometry for aesthetic or practical reasons. In the following, the construction stages and the geometry of the constructed ribs will be described coining the term “structural geometry”.

From the structural point of view, some technical terminology is essential. Transverse ribs are the main ribs in Gothic ribbed vaults. These ribs are used to bridge the nave, connecting two walls on both sides and also separating the cell units in a vault. Tiercerons function as tangential and secondary ribs in a Gothic ribbed vault and liernes are minor ribs in a complex ribs network that do not start from the main springers. The author has chosen to adopt this terminology to categorize the ribs in the analyzed system according to their function.

Observations of the vault revealed the presence of three types of ribs on the structure of the vault. The first type identified was the massive transverse ribs. These ribs bridge the width of the open space, with landings on both sides of the structure composed of a greater number of bricks as evidently serving to carry the bulk of the loads onto the piers.
According to the archival documents (Figure 5), it could be observed that four transverses labeled AF, BE, CH and DG (Figure 6) were built during the first step of the construction process.

The four transverses are two-footed landing ribs, bridging the entire width of the open space, which measures around 10 meters. Each of the transverses represents a four-centered pointed arch in front view. In Islamic architecture, multi-centered arches can be grouped into recognizable sets. The origin of four-centered pointed arches requires further study, but their structural application can be traced back to the 15th and 16th century. Here, we witnessed a famous Iranian arc profile that is colloquially known as panj-o haft.

The four transverses are thicker than the other ribs; they have cross-sections measuring around $65 \times 35$ cm and seven layers of bricks. The four transverses form two intertwined rectangles on the plan (ABEF and HCDG). These four vaults were constructed in accordance with the theoretical geometry used in their planning. These four vaults will be denoted by calling the transverses the “first layer of the ribs network” in the following.

In the second stage of construction, the vault was fortified by a network of secondary ribs (tiercerons), and fewer brick rows were used here than in the first stage of construction. The use of fewer bricks lightens the tiercerons, and these elements land on transverses on one side, and on the piers on the other side.

In certain situations, master builders may remove or trim intersected lines that occur with the theoretical geometry. For practical reasons, Us Reza trimmed the tierceron lines where they meet the transverse line. Figure 7 demonstrates that, trimming the extension of the secondary ribs (dashed lines) turns the 12-sided rosette into a square. This mean that the extensions of the tiercerons are not constructed in the finished construction.

The trimming of the lines is left to the discretion of the designer. With a practical approach, trimming the central rosette and turning it into the square is due to the small working space to execute brick groins, according to the author’s workshop experiences and learning from first-hand practitioners. Groins cannot be constructed in close proximity to each other as such an arrangement does not provide the architect with enough space to work. The elimination of the ribs reduces the weight of the structure and the amount of materials required for the construction. It seems likely that the master combined his desire to have a suitable work environment with his desire to reduce the structure’s weight and the cost of materials.

In Figure 7, the bold lines indicate the position of the tiercerons in the Kolahdouzan Dome. The tiercerons were added in the second construction phase. The dashed lines in the figure show the parts that the master chose not to include in the construction. The web between the transverses (two-footed landing ribs) and the tiercerons (single-footed landing ribs), are the tertiary ribs, landing on the transverses built in the first stage of the construction on one side, and on the tiercerons on the other side. Parts of the theoretical geometry have also been eliminated in this construction phase. The lines were trimmed and were not extended beyond the junction with the transverse and tierceron lines (Figure 7). Because these ribs are only connected to other ribs, as opposed to being connected to a springer, they will be referred to liernes in the following paragraphs.

In the current Iranian architectural discourse, a number of specialists have accepted that the ribs classified as secondary (tierceron) and tertiary (lierne) are not involved in load-bearing capacity of the vaults. They assert that they simply frame the brick shell of the dome. In other words, it is argued that the transverses are solely responsible for transferring the static loads of the cupola to the piers, with the rest elements appearing as symmetrical infill laid out on a layer of veneer. An understanding of the structural function of the elements of the vault cannot be gained without performing a dissection of the star-vault and subjecting
4.4 Architectural geometry and embedded ribs

Identifying the structural or decorative nature of the architectural components of Iranian structures has been a controversial subject for scholars, especially as the issue pertains to ribbed vaults over ceilings. In many cases, the surface of the ceiling is merely a decorative layer such as muqarnas (stalactites) and yazdi-bandī (Nazari & Hashemi Nik 2019). For example, the decays of the surface of the structure of the yazdi-bandī at the shrine of Bahau’ddin-e Naghsbandī in Bukhara has revealed the structural embedded ribs of its stellar vault, which were once covered by a yazdi-bandī pattern (Figure 9).

In other structures, the load-bearing frame is almost traceable from bottom view (Figure 11), as in the case with the architecture of the Tabriz Bazaar. However, a novel application of the embedded ribs can be observed in the cupola in Tabriz. Us Reza trimmed parts of the theoretical geometry, thereby transforming the rosette at the apex of the structure into a square, and trimming the tierceron lines in junction with liernes and transverses. For practical and aesthetic reasons, the geometry of the stellar vaults often ends with a rosette (shamseh) at the top of the structure in the Iranian stellar vault tradition. The square over the pinnacle of the vault broke with traditional ideas of aesthetics.

Here, the master’s novelty appears in his solution to the aesthetic problem. Us Reza has not displayed the entire skeleton of the vault, the key to understanding his approach in correlation between structure and architecture. He decided to use embedded ribs to hide the apex of the transverses from the field of vision of any observers. Figure 11 is a photogrammetric view of the Kolahduzan Dome, captured using a wide-angle lens, that shows the architectural geometry of the vault. It appears that the builder opted to conceal the apexes of the transverses and tiercerons by shaping a new rosette, in a larger span this time. The dashed lines in the figure indicate the positioning of the segments concealed under the brick shell of the vault. The protrusions on the outer surface of the vault are visible from the roof (Figure 11 bottom). Hiding the crowns of the transverses and tiercerons causes a rosette shape to be displayed. Thus the expected appearance for and Iranian ribbed vault is achieved. This time the rosette covers a larger span magnifying the dome larger in span.

5 THE CONSTRUCTION TECHNIQUE OF THE RIBS

The following description was made possible by the contribution of Dr. Farhad Tehrani, a close acquaintance of Us Reza, who was in charge of the Iran Cultural Heritage Center in the province of Azerbaijan, during the time of the construction of the Kolahduzan structure.

In this section the construction method of the ribs will be described, and the gypsum centering rib method, known as “lenge gachi” in Iranian traditional architectural vocabulary will be introduced. The use of this technique dates back to the Sassanid Dynasty (224-651CE) (Acre 2008, 524).

The traditional method of rib construction in Iran differs from the common traditional methods in Western architecture – what was common in Roman architecture and later revived in Renaissance architecture – where heavy, load-bearing wooden formworks were used to build arches. One of the reasons that wooden formworks are not used in Iran is due to a lack of
wood. Additionally, Iran’s climatic conditions may make wood constructions unappealing.

Iranian arches, especially the multi-centered types, have complex geometry and are difficult to draw. Therefore, they are difficult to implement without formworks. An Iranian solution to this problem is to build a gypsum centering rib, to use as a model and guide. A centering rib is a temporary wooden or gypsum structure that is built to support an arch during construction.

To build a gypsum centering rib, firstly, a full scale of the front view of the desired arch is sketched on the ground. The desired arch can be a pointed arch or of an oval arch type. The front view of the sketch is a two parallel arch with 10–18 cm space between. A mold is formed by placing the bricks on the sketched line. Sand is then poured under the mold to prevent the gypsum rib from sticking to the ground. The frame is then filled with gypsum slurry. In the past, date palm tree fibers, or bamboo stems were placed into the frame to increase the tensile strength of the gypsum centering rib. Today, plastic ropes are often used instead of tree fibers or bamboo stems. The construction of the load-bearing ribs begins after the centering is installed. One or two parallel gypsum centering rib will be set where the load-bearing transverse or tiercerons are to be built. Then, the space between the centering ribs is infilled with brickwork. Figure 12 illustrates different types of brickwork commonly used in rib construction. In order to increase the firmness and load-bearing capacity of the masonry ribs, a mixture of pitched-brickworks and radial-brickwork is used. For instance, for the Masjidi Sharif project Us Reza covered almost one sixth of the area with radial-brickwork, and the remaining area with pitched brickwork. Dr. Tehrani asserts that Us Reza has only employed pitched brickwork at the Kolahduzan, as Figure 5 indicates.

6 CONCLUSION

This research describes the Kolahduzan Dome in the old bazaar complex in Tabriz that was reconstructed in the early 1980s. While studying the dome’s stellar composition, I realized that the vault may reveal the approach to aesthetics and structure taken by an important Iranian traditional architect. Using a surveillance camera to record the points on the vault, and converting these points into AutoCAD revealed that the arch profiles matched those of four-centered pointed arches, panjo haft. The arrangement of the transverses, tiercerons and liernes match the theoretical geometry (kārbandī {12/4}). This indicates that theoretical geometry was used in this structure. The master builder expertly manipulated the theoretical geometry; he eliminated the extension of the lines in some areas. Thee manipulations satisfied the builder’s desire to reduce the weight of the structure and the quantity of construction materials used during construction. Additionally, these manipulations created ample space for the vaulting process. To distinguish the geometry of the structure from theoretical geometry, the network of the emerged ribs is called Structural Geometry. In Iranian stellar vaults, the geometry of the vaults often ends with a rosette at the top of the structure. Us Reza concealed the crown of the transverses to achieve that aesthetic ideal. We witnessed that by
hiding the crowns, the geometry of the visible ribs from the interior view is ended in a rosette, as expected. It may be stated that when a master builder, such as Reza Memaran, took a structure in hand, he followed through on every stage of the work, from the working drawings to the design of the vaults and the working out of all types on decoration and aesthetics. The geometrical approach and construction techniques adopted by the master have been identified; therefore, they can be applied in future restoration projects.

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